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CONDENSATION HEAT TRANSFER IN A MIXED FLOW REGIME

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Abstract—Experiments were performed on condensing steam within a horizontal pipe to determine a valid criterion for the flow transition from the initial annular regime to the final stratified regime. It was established that the stress ratio, $F = \tau_w/\rho_L g\delta$, determined the flow regime. Thus

$$F > 29 \quad \text{Annular}$$
$$29 \geqslant F \geqslant 5 \quad \text{Transition}$$
$$F < 5 \quad \text{Stratified}$$

describes the relevant flow regime. On this basis, a Nusselt number correlation is proposed:

$$N_{tr} = N_{an} + \frac{F-29}{24}(N_{an} - N_{str}).$$

A Nusselt number computed on the basis of this correlation exhibits errors of similar magnitude to established methods for annular and stratified flow condensation.

NOMENCLATURE

C_{pL}, specific heat at constant pressure of the liquid;
D, internal tube diameter;
f_v, fanning friction factor for vapor flow;
F, stress ratio $(\tau_w/\rho_L g\delta)$;
F_{str}, stress ratio at stratified–transitional point $(F_{str} = 5)$;
F_{an}, stress ratio at transitional–annular point $(F_{an} = 29)$;
g, gravitational acceleration;
G, mass velocity (total mass flow rate per unit total cross sectional area);
h, heat-transfer coefficient;
h_{fg}, latent heat of vaporization;
k_L, thermal conductivity of the liquid;
L, axial distance from beginning of condenser tube;
M, constant;
N, Nusselt number, (h/Dk_L);
N_{tr}, Nusselt number in transitional flow;
N_{an}, Nusselt number in annular flow;
N_{str}, Nusselt number in stratified flow;
N_{Pr}, Prandtl number of the liquid, $\mu_L C_{pL}/k_L$;
$(N_{Re})_L$, liquid Reynolds number, $(1-x)GD/\mu_L$;
$(N_{Re})_v$, vapor Reynolds number, xGD/μ_v;
T^+, specified function of δ^+ and N_{Pr};
ΔT_s, saturation temperature—tube wall temperature;
u_*, shear velocity;
x, steam quality;
X, Martinelli parameter, the ratio of pressure gradients in the liquid phase at mass velocity $(1-x)G$ to that in the vapor phase at mass velocity xG.

Greek symbols

α, void fraction;
β, semiangle subtended by draining condensate;
δ, liquid film thickness;
δ^+, nondimensional film thickness;
μ_L, dynamic viscosity of liquid;
μ_v, dynamic viscosity of vapor;
ρ_L, liquid mass density;
ρ_v, vapor mass density;
τ_w, wall shear stress;
τ_{v0}, wall shear stress if only the vapor were flowing;
ϕ_g, two-phase multiplier for shear stress in "frictional" pressure gradient.

INTRODUCTION

IT IS well known that the major difficulty in predicting heat transfer in two phase flow resides in the inability to predict the pertinent flow regimes for specified flow parameters. The standard predictive method for flow regime determination for horizontal flow is to use the Baker flow map [1]. Many other flow maps exist. In [2], for example, Soliman and Azer use a Froude number and void fraction as co-ordinates of a flow map.

The present purpose is to produce an analytic criterion for one specific flow regime transition which can then be cast in a convenient form for use in heat-transfer calculations. In particular, we are seeking heat-transfer results in the flow regime bounded by fully annular and fully stratified flow. To define our terms, we refer to Fig. 1 which is a sketch of progressive condensation inside a horizontal tube. The initially high vapor velocity in the tube maintains annular flow; low vapor velocity near the exit end of the tube does not

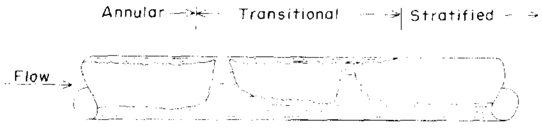

FIG. 1. Flow patterns in condensation.

support the annular flow. Condensation there is classical Nusselt film flow with some additional complications arising from the drainage of accumulated condensate along the bottom of the tube. Between these extremes an unsteady transition flow regime exists such as shown in Fig. 2 and observed experimentally using sight glasses in the apparatus of [3].

We obtained the modest L/D range by adjusting the incoming steam flow so that it was just stratified at the condenser tube exit which made it possible to assign transitional flow to the thermocouple station preceding that of the exit position.

Flow transition

A correlation of the transition between annular and stratified flow will be made in terms of the stress ratio, F, where

$$F = \frac{\text{Axial shear force}}{\text{Gravitational body force}} \equiv \frac{\tau_w}{\rho_L g \delta}. \quad (1)$$

The basic problem is to discover a convenient form

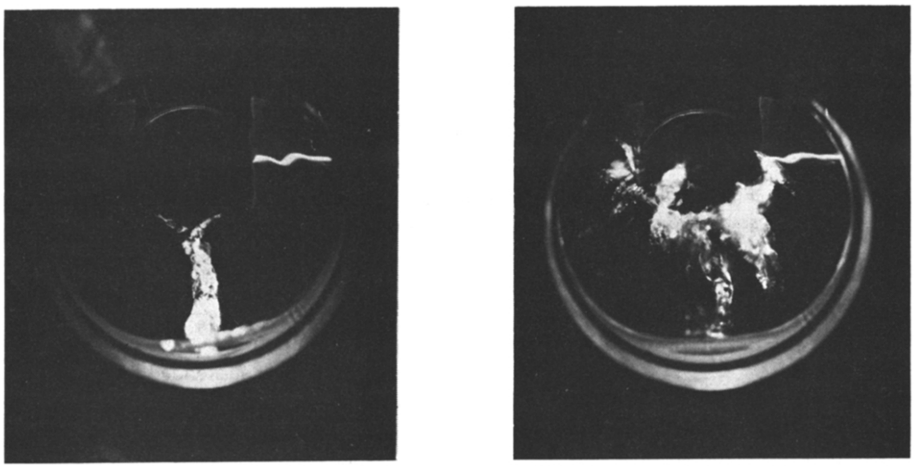

FIG. 2. Transition flow. $G = 71{\cdot}2\,\text{kg/m}^2\,\text{s}$; $x = 0{\cdot}207$; $P = 93080\,\text{N/m}^2$.

As just implied, the transition regime is dominated by effects of vapor shear and of drainage down the condenser tube walls under the influence of gravitational body forces. In the following section, we will use the ratio of these stresses as the parameter for flow regime determination. Subsequently, we will base a Nusselt number correlation on this concept arguing that the transition regime heat-transfer characteristics lie between those in annular flow and those in the stratified flow.

Data in the steam/water system have been taken from [3] and supplemented by additional measurements in the mixed flow regime and the stratified regime. The reader is directed to [3] for a description of the experimental method. The range of parameters investigated is given in Table 1.

Table 1

Range of experimental parameters

Length	2·20 m
D	12·5 mm
L/D	136–173
x at exit	0·05–0·6
pressure	2×10^4–$1{\cdot}70 \times 10^5$ N/m^2
G	12·6–145 kg/s m^2

to express τ_w and δ in terms of the accessible flow parameters of the system.

We proceed using the Martinelli 2-phase multiplier method as formulated by Wallis [4].

$$\phi_g^2 \equiv \frac{\tau_w}{\tau_{v0}} \quad (2)$$

where τ_{v0} is the wall shear stress which would exist if only the vapor were flowing at mass velocity xG. We will assume a Blasius formula for τ_{v0}.

$$\tau_{v0} = \frac{f_v x^2 G^2}{2\rho_v} \quad (3)$$

where

$$f_v = \frac{0{\cdot}079}{(N_{Re})_v^{0{\cdot}25}}. \quad (4)$$

It is, of course, normal to specify that equation (4) is valid for turbulent flow. We found, however, in the course of this investigation that we could use equation (4) to the accuracy of our observations irrespective of $(N_{Re})_v$.

Following Wallis [4], we correlate ϕ_g^2, in terms of the ratio of pressure gradients of all liquid to all vapor flow

$$\phi_g = (1 + X^{2/M})^{M/2} \quad (5)$$

where

$$X^2 = \left(\frac{\mu_L}{\mu_v}\right)^{0.25}\left(\frac{\rho_v}{\rho_L}\right)\left(\frac{1-x}{x}\right)^{1.75} \tag{6}$$

assuming turbulent flow in both phases.

The value of M is given variously as $M = 4$ (Wallis), in condensation as $M = 5.13$ [3], but in fact appears to be a slowly varying function of pressure and flow, at least in the water/steam system when the experimental correlation of the high pressure data of Boiko [5] is examined.*

To complete the expression for F we need the annular film thickness δ. Following Kosky [6], we use the shear velocity $u_* \equiv \sqrt{[\tau_w/\rho_L]}$. The non-dimensional film thickness $\delta^+ \equiv \rho_L u_* \delta/\mu_L$ is a unique function of the liquid Reynolds number $(N_{Re})_L$.

$$\delta^+ = \sqrt{\left[\frac{(N_{Re})_L}{2}\right]} \quad \text{if} \quad (N_{Re})_L \leqslant 1250 \tag{7a}$$

$$\delta^+ = 0.0504(N_{Re})_L^{0.875} \quad \text{if} \quad (N_{Re})_L > 1250. \tag{7b}$$

Combining equations (1) through (7).

$$F = \frac{(1+X^{2/M})^{3M/2}\rho_L x^3 G^3 f_v^{3/2}}{2\sqrt{2}(\rho_v\rho_L)^{3/2}\mu_L\delta^+ g}. \tag{8}$$

We compared data taken under condensation in the annular, transitional and stratified flow regimes. The flow regime map given as Fig. 3 shows that the value of F is a clear indicator of the observed flow regime.

Thus, for

$F > 29$ Annular
$29 \geqslant F \geqslant 5$ Transition
$F < 5$ Stratified

describes the relevant flow regime. For future reference, we write $F_{an} = 29$ and $F_{str} = 5$.

Heat-transfer correlation

In the fully annular regime, the heat flow through the flowing film is governed by boundary layers whose thermal resistance can be expressed by the usual Martinelli analogy [7] as modified in [3].

$$N_{an} = \frac{1}{T^+}\frac{\rho_L D C_{pL} u_*}{k_L} \tag{9}$$

where

$$T^+ = \delta^+\left(\frac{5}{\delta^+}\right)^{H(\delta^+-5)}\left[N_{Pr} + H(\delta^+-5)\ln\left\{1+N_{Pr}\left(\frac{\delta^+}{5}-1\right)\left(\frac{5}{\delta^+/5-1}\right)^{H(\delta^+-30)}\right\} + H(\delta^+-30)\ln(\delta^+/30)\right] \tag{10}$$

in which the Heaviside unit function is defined by

$$H(x-c) = 0 \quad \text{if} \quad x < c$$
$$= 1 \quad \text{if} \quad x \geqslant c.$$

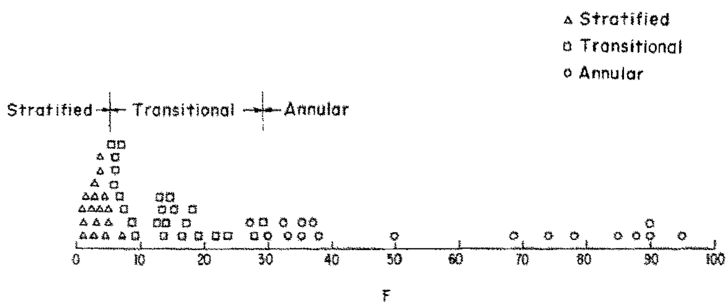

FIG. 3. Flow regime map.

By means of these relationships and equation (7) for δ^+ and equations (2)–(6) for u_*, we may calculate N_{an} for these conditions in which $F > F_{an}$. For fully stratified flow $F < F_{str}$, we may proceed analogously to Nusselt's method, e.g. [8]. We choose a simplified model over that of Rufer and Kezios [9] for this regime since their relationships are very inconvenient to use. Our simplification basically assumes that heat transfer through the accumulated condensed flow is negligible compared to the film condensation in the upper portions of the tube. If we further assume a flat draining liquid level then

$$N_{str} = \frac{0.725D}{k_L}\left[\frac{k_L^3\rho_L(\rho_L-\rho_v)h_{fg}g}{\mu_L D\Delta T_s}\left(\frac{1+\cos\beta}{2}\right)^3\right]^{1/4} \tag{11}$$

where β is the half angle subtended at the center of the tube by the chord formed at the surface of the draining film. Under the assumptions cited β can be related to the void fraction, α, by

$$\alpha = \left(1-\frac{\beta}{\pi}+\frac{\sin 2\beta}{2\pi}\right). \tag{12}$$

where α may be calculated from, for example, [10],

$$\alpha = \frac{1}{1+\dfrac{1-x}{x}\left(\dfrac{\rho_v}{\rho_L}\right)^{2/3}}. \tag{13}$$

A great simplification is achieved by noting that

$$\cos\beta \approx 2\alpha - 1 \tag{14}$$

(see Fig. 4).

*$M = 4.28 - 1.45\times10^{-7}(p - 4.76\times10^6) - 5.05\times10^{-6}(G-489)^2$
$6.86\times10^6 > p > 2.42\times10^6$ N|m^2
$489 > G > 48.9$ kg|m^2 s
[N. D. Fitzroy contributed in establishing this relationship].

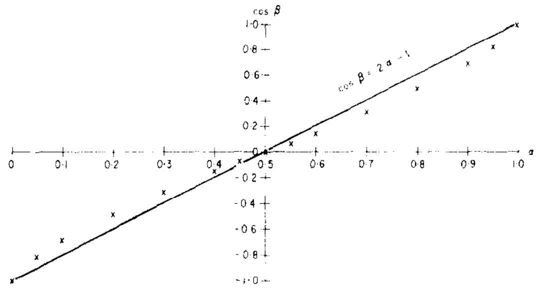

FIG. 4. Equation (12) vs linearization.

Therefore, the Nusselt number for stratified flow can be expressed as:

$$N_{str} \simeq 0.725\left[\frac{\rho_L(\rho_L-\rho_v)h_{fg}gD^3\alpha^3}{k_L\mu_L\Delta T_S}\right]^{1/4}. \quad (15)$$

We now propose that in the transition range $F_{an} > F > F_{str}$, a linear interpolation for N_{tr} between the annular and stratified Nusselt numbers will suffice as a good approximation, i.e.

$$\frac{N_{tr}-N_{an}}{N_{an}-N_{str}} = \frac{F-F_{an}}{F_{an}-F_{str}} \quad (16)$$

where N_{an}, N_{str} are calculated with aid of the relevant formulae using the local flow parameters.

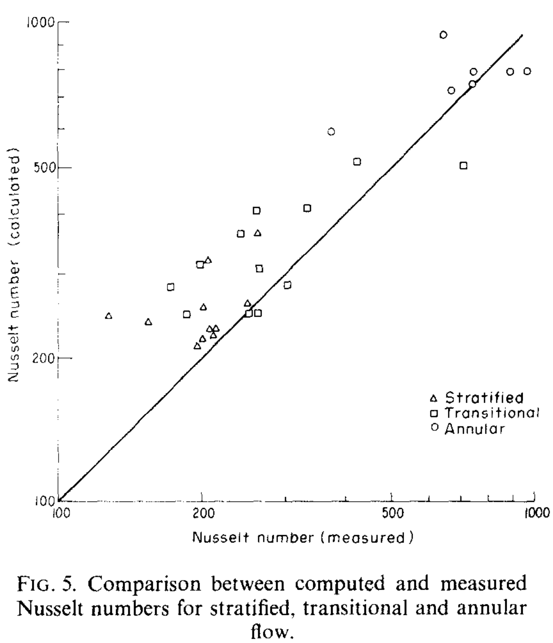

FIG. 5. Comparison between computed and measured Nusselt numbers for stratified, transitional and annular flow.

To test this hypothesis, the data of [3] and additional data in the transitional and stratified regimes were experimentally obtained and are plotted as Fig. 5. From this graph, we have computed a normalized variance for each regime by formula

$$\sigma = \sqrt{\left[\frac{1}{n}\sum_{i=1}^{n}\frac{(N_{mi}-N_{ci})^2}{N_{mi}}\right]}$$

where N_{mi}, N_{ci} are the measured and calculated Nusselt numbers for $i = 1, n$ points.

For these data, Table 2 summarizes our results.

Table 2

Regime	Normalized standard deviation
Annular	0·27
Transition	0·34
Stratified	0·37

Thus, it is clear that the linear interpretation, equation (16), is as good a fit for the transitional flow regime as for the bounding regimes for which established theories exist.

CONCLUSIONS

A single valued criterion has been established for the transition in flow regime between annular and stratified flow. This is the stress ratio F, the axial shear forces/gravitational body forces. Using steam/water data, we have measured values of F at the boundaries of the transitional flow regime as per Table 1. A caveat is in order: While our experiments did not indicate a flow regime dependence on L/D ratio, the small range of L/D actually tested may limit this conclusion to our tested range ($173 > L/D > 136$). Note also that we did not distinguish between turbulent and laminar flow as specified on a simple $(N_{Re})_v$ basis. In other words, we used equation (4) to compute f_v irrespective of the vapor Reynolds number.

The heat transfer in the mixed flow regime was correlated in terms of a Nusselt number computed as a linear function of the stress ratio, according to equation (16). It was further shown that errors in prediction of N_{tr} in this range were within errors in prediction of N_{an}, N_{str} in their respective ranges.

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TRANSFERT DE CHALEUR PAR CONDENSATION DANS UN REGIME D'ECOULEMENT MIXTE

Résumé—Des expériences ont été effectuées sur un écoulement de vapeur avec condensation dans un tube horizontal afin de déterminer un critère de transition valable depuis le régime annulaire initial jusqu'au régime stratifié final. Il a été établi que le rapport des contraintes, $F = \tau_w/\rho_L g\delta$, détermine le régime de l'écoulement. Ainsi le régime d'écoulement est caractérisé par

$F > 29$ régime annulaire
$29 \geqslant F \geqslant 5$ régime de transition
$F < 5$ régime stratifié.

Sur cette base, une expression du nombre de Nusselt est proposée:

$$N_{tr} = N_{an} + \frac{F-29}{24}(N_{an} - N_{str}).$$

Le nombre de Nusselt calculé à l'aide de cette formule présente des erreurs d'importance comparable à celles fournies par les méthodes établies pour la condensation en écoulement annulaire et stratifié.

WÄRMEÜBERGANG BEI KONDENSATION IN EINEM MISCHSTRÖMUNGSREGIME

Zusammenfassung—Zur Bestimmung der gültigen Kriterien für den Strömungsübergang von der anfänglichen Ringraumströmung zur endgültigen Schichtenströmung wurden Experimente mit kondensierendem Dampf in einem horizontalen Rohr durchgeführt. Es ergab sich, daß das Strömungsregime durch das Spannungsverhältnis $F = \tau_w/\rho_L g\delta$ bestimmt wird. Damit gilt für die Strömungsregime

$F > 29$ Ringraumströmung
$29 \geqslant F \geqslant 5$ Übergangsströmung
$F < 5$ Schichtenströmung.

Für die Nusselt-Zahl wird folgende Korrelation vorgeschlagen:

$$Nu_{tr} = Nu_{an} + \frac{F-29}{24}(Nu_{an} - Nu_{str})$$

Die mit dieser Korrelation berechneten Nusselt-Zahlen zeigen Abweichungen von etwa der gleichen Größe wie sie nach anderen Beziehungen für die Kondensation in Ringraum- und Schichtenströmung gefunden wurden.

ТЕПЛОПЕРЕНОС ПРИ КОНДЕНСАЦИИ В УСЛОВИЯХ СМЕШАННОГО РЕЖИМА ТЕЧЕНИЯ

Аннотация — Проводились эксперименты с конденсирующим паром в горизонтальной трубе в целях определения критерия применимости для переходного режима течения от начального кругового течения к конечному слоистому течению. Было установлено, что отношение напряжения, $F = \tau_w/\rho_L g\,\delta$, определяет режим течения. Таким образом,

$F > 29$, круговое
$29 \geqslant F \geqslant 5$, переходное
$F < 5$, слоистое

описывает соответствующий режим течения. Исходя из этого, предлагается соотношение числа Нуссельта. Число Нуссельта, подсчитанное на основе этого соотношения, обладает погрешностями, аналогичными для методов, разработанных для конденсации в круговых и слоистых течениях.